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High-frequency acoustic noise emission excited by laser-driven cavitation

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A high-frequency part of the acoustic noise spectrum excited by laser-driven cavitation in liquid is investigated theoretically. It is assumed that the liquid is inviscid and compressible and the surface tension may be neglected. The specific heat ratio is taken to be 5/3. It is shown that, in the first approximation, the spectral density of the acoustic energy emitted by a cavity explosion varies as the $-4/7$ power of the frequency and asymptotically decreases by ~ 3.4 dB/octave.

1. Introduction

The production of shock waves in liquids by laser radiation was reported some time ago by Carome, Moeller & Clark (1966) and Felix & Ellis (1971). Experimental studies of the acoustic signals induced in liquids by the focused beam from a Q-spoiled laser have been carried out by Vogel & Lauterborn (1988) and Vogel, Lauterborn & Timm (1989). Very intense acoustic impulses have been produced with laser pulses of less than 0.05 J total energy. The observed impulses have peak pressures of approximately 500 atm and frequency components in excess of 2400 MHz (Carome *et al.* 1966). This phenomenon is of importance in the medical fields of urology and ophthalmology, e.g. the extra corporeal shock wave lithotripter, a device for destruction of kidney stones (Coleman *et al.* 1987; Steiner 1988), in ocular surgery with pulsed lasers (Frankhauser *et al.* 1981), in crystal processing and destruction and in underwater acoustics and optics.

Chia-Lun Hu (1969) has derived a spherical fast-heating model for the laser-generated acoustic wave in which the pressure at any point and any time is expressed in terms of the input parameters. Simple models for the hydrodynamics of laser-heated exploding foils (London & Rosen 1986) and the self-similar power-driven and adiabatic expansion into vacuum (Farnsworth *et al.* 1979; Farnsworth 1980; Fabbro, Max & Fabre 1985) have been developed. Note also the studies of Khonkin & Orlov (1993), Tzuk *et al.* (1993), and Karabutov & Rudenko (1976), in which the hydrodynamics of the sudden expansion of a gas is considered. The solutions obtained give a Gaussian profile for the material density distribution. On the other hand, the dynamics of a gas bubble in a liquid is strongly dependent on the pressure of the gas in the bubble (Prosperetti, Crum & Commander 1988; Keller & Miksis 1980). This quantity must be determined from the solution of the conservation equations inside and outside the bubble. For large-amplitude bubble oscillations, a polytropic or adiabatic relation is usually used:

$$p = p_0(R_0/R)^{3\gamma}, \tag{1}$$

where p is the gas pressure in the bubble, R is the bubble radius, and γ is the specific heat ratio. The index '0' corresponds to an initial value. Note that Church (1991) supposes that a more realistic assumption is a van der Waals equation of state for a 'real' gas. Many studies have been devoted to the hydrodynamic theory of sonoluminescence (Löfstedt, Barber & Putterman 1993; Barber & Putterman 1992; Hiller, Putterman & Barber 1992) in which the light emission appears to be generated by pulsating in an acoustical field bubble.

The phenomenon of laser-excited cavitation has much in common with a strong explosion in a homogeneous medium, which represents a typical example of a self-similar flow (Zel'dovich & Raizer 1966). The gas motion in this case will be determined by two parameters: the energy of the explosion, E, and the initial density, ρ_0.

The breakdown and heating of a gas under the action of a concentrated laser beam has been studied in the past theoretically and experimentally and a vast bibliography can be found in the above-mentioned monograph of Zel'dovich & Raizer and in Vogel & Lauterborn (1988). The investigation of laser-induced cavitation is complicated in that the thermodynamic properties and phase diagram of water at pressures of the order of tens of thousands of atmospheres and at temperatures of tens of thousands of degrees are, up to now, not well known, unlike the well-studied properties of high-temperature air.

The present study was undertaken as an attempt to estimate the asymptotic behaviour of the acoustic noise spectrum induced by laser-excited cavitation in liquid.

2. Deposition of energy

The equations of continuity, momentum, energy and state are (the case of the spherical symmetry and an inviscid liquid are considered)

$$\frac{\partial \rho}{\partial t}+v\frac{\partial \rho}{\partial r}+\rho\frac{\partial v}{\partial r}+2\frac{\rho v}{r}=0, \tag{2}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0, \tag{3}$$

$$\frac{\partial}{\partial t}\left(\rho\epsilon+\frac{\rho v^2}{2}\right)=-\frac{\partial}{\partial x}\left[\rho v\left(\epsilon+\frac{v^2}{2}\right)+\rho v\right]+\rho Q, \tag{4}$$

$$p=\rho\Re T \quad \text{(for gas or plasma)}, \tag{5}$$

$$\frac{p+B}{p_0+B}=\left(\frac{\rho}{\rho_0}\right)^n \text{(Tait's equation for water)}, \tag{6}$$

where ρ is the mass density, v is the radial velocity, t is the time, r is the radial coordinate, p is the pressure, ϵ is the internal energy per unit mass, Q is the net energy input per unit mass, T is the absolute temperature, $\Re$ is the specific gas constant.

In Tait's equation, $B \approx 3000$ atm and $n \approx 7$. (The value of the gas constant is 8.3143 J mol^{-1} K.) The process of heating may be assumed isentropic because the thermal dissipation is very small even in the shock wave front. Then, the well-known relation

$$\mathrm{d}E = T\mathrm{d}s-p\mathrm{d}(1/\rho), \tag{7}$$

where E is the energy deposited, and s is the entropy, can be simplified to

$$\mathrm{d}E \approx -p\mathrm{d}(1/\rho). \tag{8}$$

Integrating this expression taking into account (5) yields

$$E = c^2 \ln \rho = c^2 \ln\left(\frac{p}{\Re T}\right), \tag{9}$$

where $c = (\mathrm{d}p/\mathrm{d}\rho)^{1/2}$ is the isothermal speed of sound. If uniform pressure in the bubble is assumed, it follows that (for unit of mass)

$$p/\Re T = \mathrm{e}^{E/c^2}. \tag{10}$$

As has been noted by Felix & Ellis (1971), simultaneously with the deposition of the laser energy, small solid impurities or dissolved gases absorb enough energy to become extremely hot and local explosions occur in the liquid. The expression for the surface temperature during the heating of a semi-infinite body by constant heat flux was obtained by Carslow & Yaeger (see Arecchi & Schulz-Dubois 1972):

$$T = \frac{2H}{k}\left(\frac{Kt}{\pi}\right)^{1/2}, \tag{11}$$

where $H = E/t$ is the applied heat flux that is constant in time and uniform across the surface of the target, k is the thermal conductivity coefficient, and K is the thermal diffusivity coefficient.

It is assumed that the known deposited laser energy can be averaged over a finite amount of time (pulse duration).

Now, from (10), the instantaneous initial pressure in the forming gas bubble can be defined as

$$p_0 = \Re\frac{2H}{k}\left(\frac{Kt}{\pi}\right)^{1/2}\mathrm{e}^{E/m\Re T} = \frac{2\Re E}{kt^{1/2}}\left(\frac{K}{\pi}\right)^{1/2}\exp\left[\frac{Kt^{1/2}}{2m\Re}\left(\frac{K}{\pi}\right)^{-1/2}\right], \tag{12}$$

where m is the mass of liquid transformed into the plasma state.

This mass may be approximately found from an expression for the quantity of heat that is transferred to the bubble:

$$E = \frac{4\pi}{3}LR_0^3\rho', \tag{13}$$

where L is the latent heat of vaporization and ionization (up to complete dissociation), ρ' is the plasma density, and R_0 is the initial bubble radius. The changes in L occurring at the time of the temperature rise are not taken into account.

3. Oscillations in compressible liquid

The Rayleigh–Plesset equation is employed to describe the motion of a gas-filled bubble in a compressible liquid:

$$R\ddot{R}\left(1-\frac{2\dot{R}}{c}\right)+\frac{3}{2}\dot{R}^2\left(1-\frac{4}{3}\frac{\dot{R}}{c}\right)-p\frac{(R,t)}{\rho_L}+\frac{p_\infty}{\rho_L} = 0, \tag{14}$$

where R is the bubble radius, p_∞ is the ambient pressure, ρ_L is the liquid mass density, and a dot denotes differentiation with respect to time.

The boundary condition at the fluid–gas interface may be expressed in accordance with (1) as

$$p(R,t) = p_0(R_0/R)^{3\gamma}, \tag{15}$$

where γ is the isentropic exponent for the ideal gas assumed.

According to Fabbro *et al.* (1985), the value $\gamma = 5/3$ may be used and (15) takes the form

$$p(R,t) = p_0(R_0/R)^5. \tag{16}$$

Substituting $dR/dt = \dot{R} = U$, with the assumption that $p \gg p_\infty$, in (14) gives

$$RU\frac{dU}{dt}\left(1-\frac{2U}{c}\right)+\frac{3}{2}U^2\left(1-\frac{4}{3}\frac{U}{c}\right) = \frac{p_0}{\rho_L}\left(\frac{R_0}{R}\right)^5, \tag{17}$$

and multiplying both sides by $2R^2$ yields

$$\frac{d}{dR}\left[R^3U^2\left(1-\frac{4}{3}\frac{U}{c}\right)\right] = \frac{p_0}{\rho_L}\frac{2R_0^5}{R^3}. \tag{18}$$

After integrating, (18) becomes

$$U^2\left(1-\frac{4}{3}\frac{U}{c}\right)+\frac{p_0}{\rho_L}\left(\frac{R_0}{R}\right)^5 = 0. \tag{19}$$

(If $\gamma = 4/3$ is taken, then (19) is written as

$$U^2\left(1-\frac{4}{3}\frac{U}{c}\right)+\frac{p_0}{\rho_L}\left(\frac{R_0}{R}\right)^4 = 0,$$

and for $\gamma = 1$ this equation takes the form

$$U^2\left(1-\frac{4}{3}\frac{U}{c}\right)+\frac{p_0}{\rho_L}\left(\frac{R_0}{R}\right)^3 = 0.$$

In the work that follows, $\gamma = 5/3$ is used, as mentioned above, and the influence of this parameter is not considered.)

The nonlinear differential equation (19) can, in principle, be integrated in quadratures but the solution is obtained in a parametric form. However, it can be simplified under the assumption that the bubble wall velocity after the laser pulse is approximately equal to the speed of sound in the liquid. This assumption allows a very simple equation to be obtained that can easily be integrated. It seems well founded because both optical breakdown and bubble collapse lead to an extremely high temperature and pressure within a very small volume of not more than 100 μm diameter and the laser pulse duration is 40–60 ns (Vogel & Lauterborn 1988, p. 726); then, (19) can be written as

$$U = \left(\frac{3p_0}{\rho_L}\right)^{1/2}\left(\frac{R_0}{R}\right)^{5/2}. \tag{20}$$

Substituting into (20) the initial pressure value from (12) gives

$$\frac{dR}{dt} = \frac{6\Re E}{k}\left(\frac{K}{\pi}\right)^{1/4} t^{-1/4}\rho_L^{-1/2}R_0^{5/2}R^{-5/2}\exp\left[\frac{Kt^{1/2}}{4m\Re}\left(\frac{K}{\pi}\right)^{-1/2}\right]. \tag{21}$$

Equation (21) may now be solved for the radius R:

$$R^{7/2} = \frac{7}{2}aR_0^{5/2}\int_0^t t^{-1/4}\,e^{\beta t^{1/2}}\,dt, \tag{22}$$

where, for brevity, the following values are introduced:

$$a = \frac{6\Re E}{k}\left(\frac{K}{\pi}\right)^{1/4} \rho_L^{-1/2}, \tag{23}$$

$$\beta = \frac{(\pi K)^{1/2}}{4m\Re}. \tag{24}$$

Using the substitution $t = u^2$, the integral in (22) may be expressed in terms of incomplete gamma functions (Gradshtein & Ryzhik 1965) and then the radius–time relation will be

$$R = 7^{2/7} a^{2/7} R_0^{5/7} \left\{ \left(\tfrac{1}{2}\rho_0\right)^{-3/2} \gamma\left(\tfrac{3}{2}, -\tfrac{1}{2}\beta t\right) \right\}^{2/7}, \tag{25}$$

where $\gamma(\alpha, \chi)$ is the incomplete gamma function.

If the expansion (Gradshtein & Ryzhik 1965)

$$\gamma(\alpha, \chi) = \sum_{n=0}^{\infty} \frac{(-1)^n \chi^{\alpha+n}}{n!(\alpha+n)}$$

is employed, and only the first term is retained, then the bubble radius may be found in the form

$$R \cong \left(\frac{14}{3}\right)^{2/7} a^{2/7} R_0^{5/7} \beta^{-3/7} t^{3/7} = At^{3/7}, \tag{26}$$

where A is a numerical coefficient that can be expressed in terms of a and β defined by (23) and (24). Then,

$$\dot{R} = \frac{3}{7} A t^{-4/7} \quad \text{and} \quad \ddot{R} = -\frac{12}{49} A t^{-11/7}.$$

This dependence of radius on time differs from the results described by Hsieh (1965) in his comprehensive systematic survey of analytical aspects of bubble dynamics where these relationships are $R \sim t^{3/2}$ and $R \sim t^{2/3}$ for incompressible and compressible liquids respectively.

For the velocity and pressure fields, it should be noted that the Gilmore (1952) solution for the determination of the bubble wall velocity based on the Kirkwood & Bethe hypothesis (Cole 1948) and giving good results in the case of bubble collapse (Hsieh 1965, p. 997), is inapplicable to the calculation of the bubble growth. This is because for bubble growth the denominator of the integrand in the expression for the motion of the bubble wall is equal to zero once or several times within the limits of integration, which leads to the divergence of the integral on the right-hand side of Hsieh's (1965) equation (118) p. 997.

Under these circumstances, one can suppose that the quasi-acoustic approximation (Trilling 1952) may be used for determination of the acoustic adiabatic pressure. Within this approximation, the flow potential is derived from the solution of the wave equation, while the term $\frac{1}{2}v_r^2 = \frac{1}{2}(\partial\phi/\partial r)^2$ contained in the momentum equation is preserved. With this assumption, the Lagrange–Cauchy integral, neglecting the terms of the order of $1/r^2$ at large distances from a bubble (Fitzpatric & Strasberg 1958), gives

$$p_s\left(t - \frac{r-R}{c}\right) = \frac{\rho_L R}{2r}\left[R\ddot{R}\left(t - \frac{r-R}{c}\right) + 2\dot{R}^2\left(t - \frac{r-R}{c}\right)\right]. \tag{27}$$

where p_s is the acoustic pressure.

In the derivation of this expression, the density, ρ_L, has been taken as constant, which can be done only in the case of small oscillations when the term $v_r^2/2$ is omitted. The changes in density and squared velocity have the same order of magnitude, therefore it is not logical to keep only one of them. However, in the quasi-acoustic approximation, the velocities have to be assumed so small that the velocity potential will satisfy the acoustic equation $[\partial/\partial t + c\,\partial/\partial r](r\phi) = 0$, and the momentum equation can be written as

$$\frac{\partial v_r}{\partial t} + v_r\left(\frac{\partial v_r}{\partial r}\right) + \frac{1}{\rho_L}\left(\frac{\partial p}{\partial r}\right) = 0.$$

Nevertheless, when the acoustic equation is derived, the momentum equation is written as

$$\frac{\partial v_r}{\partial t} + \frac{1}{\rho_L}\left(\frac{\partial p}{\partial r}\right) = 0.$$

It is an internal contradiction of the quasi-acoustic approximation when the value of the bubble wall velocity is not limited, but only such an approach enables a relatively simple result to be obtained. As has been shown by Gilmore (1952) for the case of bubble collapse, the results of the quasi-acoustic approximation are close enough to the results obtained when the Kirkwood & Bethe hypothesis or even the numerical solution of exact equations are used, if the Mach numbers of the bubble boundary do not exceed unity by too much.

Substituting (26) into (27) gives the acoustic pressure as a function of time:

$$p_s = \frac{\rho_L}{r} A^3 \frac{6}{49} t^{-5/7}. \tag{28}$$

The total acoustic energy emitted by the bubble (Cole 1948) is determined as

$$E_{ac} = \frac{4\pi r^2}{\rho_L c} \int_0^t p_s^2 \,\mathrm{d}t. \tag{29}$$

After substituting p_s from (28) into (29) and carrying out the elementary integration, the total acoustic energy is

$$E_{ac} = \frac{4\pi\rho_L}{c} \frac{7}{10} A^6 \left(\frac{6}{49}\right)^2 t^{-3/7}. \tag{30}$$

On the other hand, the acoustic energy emitted may be also defined as

$$E_{ac} = \int_0^\infty S \,\mathrm{d}f, \tag{31}$$

where S is the spectral density, and f is the frequency. Comparing the right-hand sides of (30) and (31) and taking into account that $f = 1/t$, gives

$$\lim_{f\to\infty} \int_0^f S(f)\,\mathrm{d}f = \frac{\rho_L^2}{r^2} \frac{7}{10}\left(\frac{6}{49}\right)^2 A^6 f^{3/7}, \tag{32}$$

and, after differentiating, yields the expression for the spectral density

$$S = \frac{3}{10}\left(\frac{6}{49}\right)^2 \frac{\rho_L^2}{r^2} A^6 f^{-4/7}. \tag{33}$$

4. Conclusions

The theoretical analysis of the frequency noise spectrum is subjected to several rough approximations that restrict the range of its applicability. However, the theoretical results allow the high-frequency part of the acoustic spectrum to be evaluated and it may be helpful for comparison with experimental data. As can be seen from (33), the spectral density of the acoustic energy varies with frequency as the $-4/7$ power of the frequency and asymptotically decreases by $20\log(2^{-4/7}) \approx 3.4$ dB/octave. As has been noted by Mellen (1954), the theory gives $S \sim f^{-2/5}$ for an incompressible liquid in the case of a bubble implosion while Benjamin (1958) and Esipov & Naugol'nykh (1972) obtained the relationship $S \sim f^{-2}$ that shows the existence of shock waves. The slow fall of the spectral density with frequency obtained here may be because of the use of the Rayleigh–Plesset equation because, as noted by Kumar & Brennen (1993), the complex processes of bubble growth and collapse are not quite adequately modelled by this equation.

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